Progression in Calculations



Addition

Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Combining two parts to make a whole: part-whole model	Use objects to add two numbers together as a group or in a bar.	Use pictures to add two numbers together as a group or in a bar.	3 + 2 = 5 9 = 8 + 1 Use the part-part whole diagram as shown or a bar representation to move into the abstract.	Progression taken from Additive Facts Guidance. In order: Adding 1 (e.g. 7 + 1 and 1 + 7) Doubles of numbers to 5 (e.g. 4 + 4) Adding 2 (e.g. 4 + 2 and 2 + 4) Number bonds to 10 (e.g. 8 + 2 and 2 + 8) Alongside: Partitioning 2, 3, 4, 5, 6 and 10

Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Starting at the bigger number and counting on.	Start with the larger number and then count on the smaller number 1 by 1 to find the answer. Ten frame activities to help become fluent. Moving on to spotting and applying known mental recall facts to solve calculations.	Start at the larger number on the number line and count on in ones or in one jump. Encourage and discuss the use of mental recall facts, to find the answer at all times.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer. Moving on to spotting and applying known mental recall facts to solve calculations. Eg: $2 + 5 = 7$ $12 + 5 = 17$ OR $5 = 2 + 3$ $12 + 3 = 15$ $15 + 2 = 17$	Adding 10 to a number (e.g. 5 + 10 and 10 + 5) Adding 0 to a number (e.g. 3 + 0 and 0 + 3) Near doubles (e.g. 3 + 4 and 4 + 3) The ones without a family! 5 + 3, 3 + 5, 6 + 3, 3 + 6 Alongside: Partitioning 7, 8 and 9

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Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Regrouping to make 10.	Start with the bigger number and partition the smaller number to make 10. Spotting and applying known mental recall facts to 10, to solve calculations.	Use pictures or a number line. Rename or partition the smaller number to make 10. 9+5=14 10 4	7 + 4 = 11 If I am at seven, how many more do I need to make 10? Partition the 4 into 3 and 1 $7 + 3 = 10$ $10 + 1 = 11$	Doubles of numbers to 10 (e.g. 7 + 7) Near doubles (e.g. 5 + 6 and 6 +5) Bridging (e.g. 8 + 4 and 4 + 8) Compensating (e.g. 9 + 4 = 10 + 4 -1)

Progression in Calculations

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Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Adding three single digits	7 + 2 + 3 = 12 Put 7 and 3 together to make 10. Add on 2. (Make 10 with 2 of the digits (if possible) then add on the third digit.) Exploring other methods involving regrouping eg: 9 + 3 + 2 = 14 Put 3 and 2 together to make 5. Group with 9 using previous strategy.	Find bonds to 10; rearrange calculation if necessary. $ \begin{array}{cccccccccccccccccccccccccccccccccc$	Combine the two numbers that make 10 and then add on the third number. Find bonds to 10 within calculations. Look for patterns and use partitioning and known additive facts to make calculation simpler.	To become mentally fluent children should look for patterns and use partitioning and known additive facts to make calculation simpler.

Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Partitioning with no regrouping/ renaming	Numicon $24 + 13 = 37$ Other resources can be used such as Base Ten.	24 + 13 = 37 $24 + 13 = 37$ $20 + 10 = 30$ $4 + 3 = 7$ $30 + 7 = 37$ Jotting alongside use of apparatus.	$ \begin{array}{r} 20 + 4 \\ 10 + 3 \\ \hline 30 + 7 = 37 \end{array} $	Mentally partition into tens and units then add and recombine/regroup.

Progression in Calculations

Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Partitioning with recombining/ regrouping	Numicon $28 + 39 = 67$ Other resources can be used such as Base Ten.	Partitioning 28 + 39 = 67 28 + 39 1	28 = 20 + 8 39 = 30 + 9 50 + 17 = 67	Mentally partition into tens and units then add and recombine/regroup, including 'exchanging' units for tens.

Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Column method - no regrouping.	Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.	After practically using the base 10 blocks and place value counters, children can draw the counters/dienes to help them to solve additions if required.	It is important that children understand the expanded method of column addition before they move onto the formal written method. This also applies to the method below. Calculations 21 + 42 = 21 + 42	Mentally partition into hundreds, tens and units then add and recombine/regroup.

Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Column method - regrouping	Add up the units and exchange 10 ones for one 10. Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.	Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding. 7 1 5 1	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. $ \begin{array}{cccccccccccccccccccccccccccccccccc$	Mentally partition into hundreds, tens and units then add and recombine/regroup, including 'exchanging' units for tens or tens for hundreds.

Progression in Calculations



Subtraction

Objective and Strategies	Concrete	Pictorial	Abstract	Mental
	Use physical objects, counters, cubes etc to show how objects can be taken away.	Pictorial Cross out drawn objects to show what has been taken away.	Abstract $6 - 2 = 4$ $15 - 3 = 12$	Progression taken from Additive Facts Guidance. In order: Adding 1 (e.g. 7 + 1 and 1 + 7) Both additions equal 8. Children should apply what they know by using the inverse calculation. e.g. 8 - 1 = 7 and 8 - 7 = 1 Doubles of numbers to 5 (e.g. 4 + 4) Adding 2 (e.g. 4 + 2 and 2 + 4) Number bonds to 10 (e.g. 8 + 2 and 2 + 8)
				Alongside: Partitioning 2, 3, 4, 5, 6 and 10

Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Counting back	Use counters and move them away from the group as you take them away counting backwards as you go. $12 - 4 = 8$	Count back on a number line or number track 9 10 11 12 13 14 15 Start at the bigger number and count back the smaller number showing the jumps on the number line. This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.	

Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Strategies Find the difference	Compare amounts and objects to find the difference. Use cubes to build towers or make bars to find the difference Use basic bar models with items to find the difference	Count on to find the difference Comparison Bar Models Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them. 13 ? Lisa Sister 22 Draw bars to find the difference between 2 numbers.	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.	

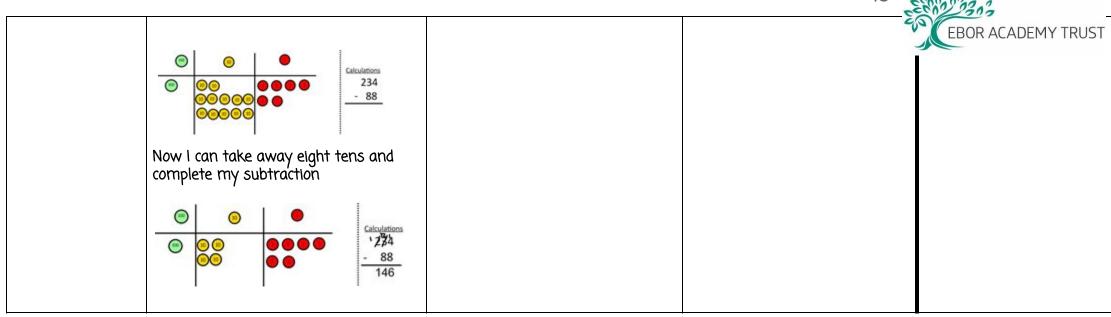
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Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Part Part Whole model	Link to additionuse the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model.	Move to using numbers within the part whole model.	



Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Make 10	Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 - 8 = How many do we take off to reach the next 10? How many do we have left to take off?	

Objective and Strategies	Concrete		Pictorial	Abstract	Mental
Column method - no regrouping	to me the beautiful the second th	n take smaller nber	Salculations 54 -22 -32	47-24=23 - <u>20+4</u> -20+3	
	Show how you partition numbers to subtract. Again make the larger number first.	7 4 30 6 - 10 4 20 2	Draw the Base 10 or place value counters alongside the written calculation to help to show working.	This will lead to a clear written column subtraction.	

Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Column method - regrouping	Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters Calculations 234 88 Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones. Now I can subtract my ones. Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.	Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make. Vhen confident, children can find their own way to record the exchange/regrouping. Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup. Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.	Children can start their formal written method by partitioning the number into clear place value columns. Moving forward the children use a more compact method. This will lead to an understanding of subtracting any number including decimals. $ \begin{array}{cccccccccccccccccccccccccccccccccc$	



<u>Progression in Calculations</u>

Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Make, find and see equal groups	Use different objects to make, find and see equal groups. e.g. take 4 plates and place an equal group on each.	Representing and finding equal groups using pictorial methods. Which activity has groups of 3? Or groups of 4?	Link pictorial representations to abstract examples using names and numbers without mentioning of operational symbol (x) e.g. Draw circles to group. 3 fives 8 twos 6 threes 5 fours	Count in equal groups according to year group expectations.

Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Doubling	Use practical activities to show how to double a number. double 4 is 8 4×2=8	Draw pictures to show how to double a number. Double 4 is 8	Partition a number and then double each part before recombining it back together. 16 10 10 10 12 20 12	Multiplying numbers by 2. Using addition of single digit, two-digit and three-digit numbers depending on year group expectations.

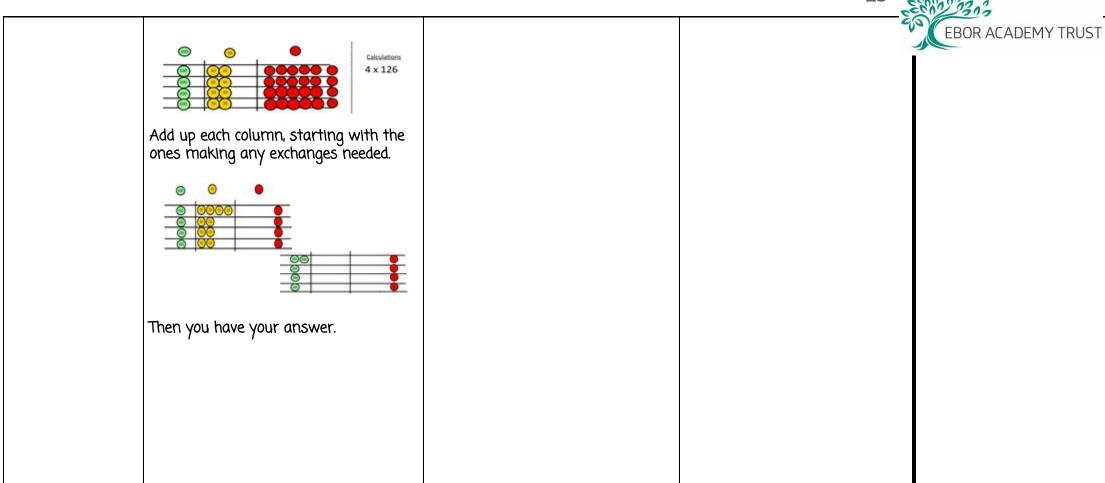
Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Counting in multiples	Count in multiples supported by concrete objects in equal groups. e.g. solving 6 x 4 by drawing / making 6 groups of 4 and skip counting to find the total. one two three 4s four 4s	Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30	Phase 1: Skip counting in numbers relating to the year group expectations. This should be forwards and backwards.

Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Repeated addition	Use different objects to add equal groups. 5+5+5+5+5+5+5+5+5+5+5+5+5+5+5+5+5+5+5	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there? 2 add 2 add 2 equals 6 5+5+5=15 Children can record this as a bar model: The dotted line shows the unknown quantity. Children could then replace the question mark with the number 40. 6 3s 3+3+3+3+3+3=18 6 × 3 = 18 Factor Factor Product Read:6 times 3 equals 18	Write addition sentences to describe objects and pictures. Children can then record this onto an empty number line: $5+5+5+5+5+5+5+5+5=40$ $0 5 10 15 20 25 30 35 40$ $8 \times 5 = 40$ $2+2+2=3 \times 2$	Relate repeated addition to multiplicative facts. Phase 2: Deriving answers using reasoning strategies based on known facts. e.g. solving 6 x 4 by thinking that 5 x 4 = 20 and adding one more group of 4.



Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Arrays-showing commutative multiplication	Create arrays using concrete apparatus to show multiplication sentences.	Draw arrays in different rotations to find commutative multiplication sentences. 4×2=8 2×4=8 2×4=8 4×2=8 1×3=8 1×	Use an array to write multiplication sentences and reinforce repeated addition. 5+5+5=15 3+3+3+3+3=15 5 x 3 = 15 3 x 5 = 15	Phase 3: Instant recall - efficient production of the answers - knowing the 6 x 4 = 24. Children should be able to explain this if asked.

Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Grid Method	Show the link with arrays to first introduce the grid method. 4 rows of 10 4 rows of 3 Move on to using Base 10 to move towards a more compact method. 4 rows of 13 Move on to place value counters to show how we are finding groups of a number. Ve are multiplying by 4 so we need 4 rows. Fill each row with 126.	Children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. X 30 5 7 210 35	Children to use knowledge of multiplication facts to increase number sizes e.g. x 10, 100, x 1000 and ÷ 10, ÷ 100, ÷ 1000. e.g. if I know 6 x 4 = 24, then I also know 60 x 4 = 240, or 6 x 400 = 2,400.





Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication. It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.	Start with long multiplication, reminding the children about lining up their numbers clearly in columns. Use as an opportunity to reinforce understanding of place value. If it helps, children can write out what they are solving next to their answer. 32	Applying knowledge of multiplication tables and using instant recall for efficiency.



Progression in Calculations

Objective and Strategies	Concrete	Pictorial	Abstract	Mental
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			x 18 13420	
			10736	
			24156	

Division

Progression in Calculations

Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Sharing objects equally	The counters are shared (individually moved, one by one) equally into two groups in this example. I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities. 8 ÷ 2 = 4 Sharing When 12 cookies are divided evenly among 3 people, 1 child gets 4 cookies. This is written with the math sentence below: 12 ÷ 3 = 4 "Tweelve divided by three equals four"	Share 9 buns between three people. $9 \div 3 = 3$	Applying inverse of year group multiplication facts.



Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Division as grouping	Divide quantities into equal groups one group (not individually) at a time. Use cubes, counters, objects or place value counters to aid understanding. 96 + 3 = 32 96 + 3 = 32 Ten divided into two equal groups	Use a number line to show jumps in groups. The number of jumps equals the number of groups. O 1 2 3 4 5 6 7 8 9 10 11 12 Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 20 20 20 20 20 20 20 20 20 2	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group? This can then be recorded on an empty number line: 28 - 7 - 7 - 7 - 7 = 0 4 groups of 7 are in 28.	Using understanding of repeated subtraction to find how many groups of a number are in the whole.

Objective and Strategies	Concrete	Pictorial	Abstract	Mental
	How many 5's in 40? Children can use Cuisenaire or Numicon to work this out using grouping Children can then take the Cuisenaire from the rod track and rearrange it into an array.	Children should be taught to represent this as a bar model:		



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Objective and Strategies	Concrete	Pictorial	Abstract	Mental
			Dod the transport of country transfers	
Division			Find the inverse of multiplication and division sentences by	
within			creating four linking number sentences.	
arrays	00000			
	The same of the sa		$7 \times 4 = 28$	
	Link division to multiplication by creating an array and thinking about	Draw an array and use lines to	$4 \times 7 = 28$	
	creating an array and thinking about the number sentences that can be created.	split the array into groups to make multiplication and division	28 ÷ 7 = 4	
	createa.	sentences.		
	$15 \div 3 = 5$		28 ÷ 4 = 7	
	$5 \times 3 = 15$			
	$15 \div 5 = 3$			
	$3 \times 5 = 15$			

Progression in Calculations

Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over.	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. Draw dots and group them to divide an amount and clearly show a remainder. The second of t	Complete written divisions and show the remainder using r. 29 + 8 = 3 REMAINDER 5 1	

Objective and Strategies	Concrete	Pictorial	Abstract	Mental
Short division	Tens Units 3 2 3 0 0 0 0 0 Use place value counters to divide using the bus stop method alongside. 42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups. Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder. $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	

Objective and Strategies	Concrete	Pictorial	Abstract	Mental
	We exchange this ten for ten ones and then share the ones equally among the groups.			
	We look how much is in 1 group so the answer is 14.			