### Objective and Concrete **Pictorial** Abstract Strategy Doubling Write addition equations using the same Draw pictures to show how to double Use practical activities with manipulatives numbers addend to equal the total. including cubes and Numicon to When doubling you always get an even demonstrate doubling number. Missing boxes for challenge. 4 + 4 = 8Counting in multiples of 2s, 5s and 10s. Count objects already grouped in 2s, 5s, 10s. Count in multiples of a number aloud. Write down missing numbers in a sequence. Forwards/backwards. 2s up to 20 5s up to 50 10s up to 100 Children group objects pictorially to find Making equal Make equal groups from a given out how many altogether by counting in groups and amount (2s, 5s, 10s separately) sequences of 2, 5 and 10. counting the total Count in sequence to find the total.

Multiplication

Recording numerals as written words and numbers when gathering the total number.

8

Double 4 is 8

12

80

e.g sixteen / 16 fifty/ 50 thirty five/ 35

8 = 4 + 4

4 + = 8

### Objective and Concrete **Pictorial Abstract** Strategy Represent equal writing expressions groups as repeated ı 3+3+3+3+3+3+3 addition eg. 2 + 2 + 2 + 2Multiplication Equal groups/redistributing Introducing x symbol. sharing objects into equal groups and recognising 2 groups of 3. (3x 2) (2 x 3) unequal groups. There are equal groups of eggs. There are \_\_\_\_ eggs in each group. There are \_\_\_ groups of \_\_\_. Represent equal 3 + 3 $2 \times 3$ groups as 3 + 3 + 3Multiplication Share 2 $4 \times 3$

 $\times 3$ 

 $4 \times 3$ 

### Objective and **Pictorial** Concrete Abstract strategy Two times table $1 \times 2 = 2$ $2 \times 2 = 4$ Adjacent multiples $3 \times 2 = 6$ $8 \times 2 =$ factor pairs written × 10 10 pencils 10 pencils 10 pencils 10 pencils in any order. 20 Repeated for 10s and 5s times tables. $0 \times 5 = 0$ $0 \times 10 = 0$ The relationship $1 \times 5 = 5$ $1 \times 10 = 10$ $2 \times 5 = 10$ $2 \times 10 = 20$ between the 5 and $3 \times 5 = 15$ $3 \times 10 = 30$ 10 times table. $4 \times 5 = 20$ $4 \times 10 = 40$ $5 \times 5 = 25$ 5 × 10 = 50 $6 \times 5 = 30$ $6 \times 10 = 60$ $7 \times 5 = 35$ $7 \times 10 = 70$ $8 \times 5 = 40$ $8 \times 10 = 80$ $9 \times 5 = 45$ $9 \times 10 = 90$ $10 \times 5 = 50$ $10 \times 10 = 100$ $11 \times 5 = 55$ 11 × 10 = 110 $12 \times 5 = 60$ 12 × 10 = 120 Multiplication is $7 \times 2 = 14$ commutative (arrays) $2 \times 7 = 14$

?

5

5

 $2 \times 5$ 

Multiplication

Objective and Strategy	Concrete	Pictorial	Abstract
Doubling numbers  Double two digit numbers  Doubling - (x2)		double 35 = 70  10  double 30 + double 5  60 + 10 = 70	Double 10 Double 1 10 + 10 = 20 1 + 2 = 2 Double 11 = 22

## Y2 Multiplication

Objective and strategy	Concrete	Pictorial	Abstract
Recall multiplication facts, and corresponding division facts, in the 10, 5, 2 times table from the previous year.  Learn the 4 and 8 multiplication tables, and recognise products in these multiplication tables as multiples of the corresponding number.	See Y2		4 × 0 = 0       8 × 0 = 0         4 × 1 = 4       8 × 1 = 8         4 × 2 = 8       8 × 2 = 16         4 × 3 = 12       8 × 3 = 24         4 × 4 = 16       8 × 4 = 32         4 × 5 = 20       8 × 5 = 40         4 × 6 = 24       8 × 6 = 48         4 × 7 = 28       8 × 7 = 56         4 × 8 = 32       8 × 8 = 64         4 × 9 = 36       8 × 9 = 72         4 × 10 = 40       8 × 11 = 88         4 × 11 = 44       8 × 11 = 88         4 × 12 = 48       8 × 12 = 96

# Y3 Multiplication



4 rows

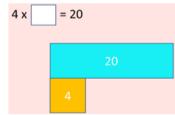
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Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.

$$24 \times 3 = 72$$

X	20	4
3	8	12

Use bar models to explore missing numbers.



Abstract

Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

×	20	5
8	160	40

## 13 Multiplication

Objective and strategy	Concrete	Pictorial	Abstract
3, 6 and 9, and the relationship between them	There are 12 wheels. $4 \times 3 = 12$ $3 \times 4 = 12$ $6  6  6  6  6  6$ $3  3  3  3  3  3  3  3  3  3 $	Draw a picture like this to show $7 \times 3 = 21$ .	$6 \times 3 = 18$ $3 \times 2 = 6$ $X = 2$ $6 \times 3 = 18$ $3 \times 2 = 6$ $0 \times 2 = 12$
Recall all times tables up to 12 x 12			

# Y4 Multiplication

### Objective & **Pictorial Abstract** Concre Strategy te Start with multiplying by one Children can represent their Show the links with arrays to first Grid method introduce the arid method. digit numbers and showing work with place value counters 4 rows of 10 the clear addition alongside recap from year in a way that they understand. 4 rows of 3 the grid. 3 for 2 digits x They can draw the counters Move onto base ten to move towards a more compact method. 1diqit using colours to show different 30 5 × amounts or just use the circles 7 210 35 in the different columns to show Move to Move on to place value counters to show how we are finding arouns of a number their thinking as shown below. We are multiplying by 4 so we need 4 210 + 35 = 245multiplying 3 digit numbers by 0000 00 00 1diqit. (Year 4 0000 Add up each column starting with the 00 ones making any exchanges needed. 60 expectation) Column 327 300 Children can continue to be 1200 80 28 Multiplication supported by place value The grid method may be counters at the stage of 28 used to show how this relates multiplication. This initially Begin by done where there is no to a formal written method. 80 $321 \times 2 = 642$ multiplying a 2 1200 digit number by 1308 a 1 digit number 8 - 60 = 480 327 This may 180 -8= (472) and progress to lead to a Bar modelling and number compact a 3 digit number lines can support learners method. when solving problems with b It is important at this stage multiplication alongside the

formal written methods.

that they always multiply the

ones first. The corresponding multiplication is modelled

<del>alongside.</del>

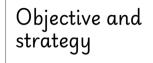
Objective & Strategy	Concrete	Pictorial	Abstract
Column Multiplication for up to 4 digits x 1 or 2 digit.	Hundreds Tens Ones  It is important at this stage that they always multiply the ones first.  Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642	x 300 20 7 4 1200 80 28	327  x 4  28  80  1200  1308  This will lead to a compact method.
Column multiplication	Manipulatives may still be used with the corresponding long multiplication modelled alongside.	Continue to use bar modelling to support problem solving	1 8 18 x 3 on the first row  x 1 3 (8 x 3 = 24, carrying the 2 for 20, then 1 x 3)  18 x 10 on the 2nd row.  Show multiplying by  x 1 6 7 4 0 4 (1234 x 6) 1 2 3 4 0 (1234 x 10) zero in ones first.

### Y5 Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Multiply numbers up to 4d x 1d using short multiplication .  Multiply numbers up to 4d x 2d using long			Ensure children remember to carry tens across to the next column and add  3 1 2 6 them onto  2 8 their answer
Multiplication  Multiplying decimals up to 2 decimal places by a single digit.			Remind children to utilize a place holder to represent the 10s in the multiplier  3 · 1 9  × 8  2 5 · 5 2  Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer.

### Y6 Multiplication

Objective and strategy	Concrete	Pictorial	Abstract	
sharing into equal groups (links to multiplication) using amounts that link to 2, 5 and 10 amounts.  Recognising unequal groups	Provide a variety of objects to share into equal groups		"6 groups of 2" "12 shared between 6 is 2" Children not to use X OR ÷ symbol.	
Halving sharing a number into groups of 2	Share objects eg. cubes/counters into groups of 2	Begin with shapes to recognise 2 equal parts.  Children may also draw hoops to record pictorially.	Children not to use ½ fraction in Year 1.  Half of 8 is  Half of 12 is	ivision
Quarters sharing a number into groups of 4	Share objects eg. cubes/counters into groups of 4.		Children not to use ¼ fraction in Year 1.  A quarter of 8 is A quarter of 12 is	



Concrete

Pictorial

**Abstract** 

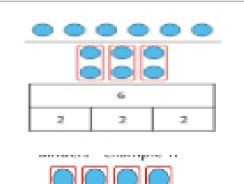
sharing objects

sometimes with a remainder

Keep groups of numbers within 2, 5 and 10.

Use real objects





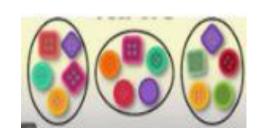
Knowledge of repeated addition and multiplication link

 $6 = 2 \times 3$ 

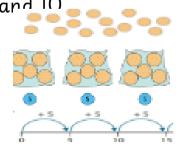
Representing quotitive division

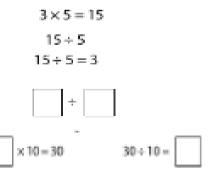
Introduction of division symbol

Division equations groups of 2, 5 and



Sharing into groups and then begin to skip count in 2, 5 and 10



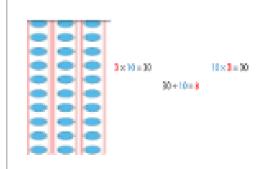


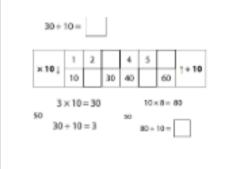
Division using known facts rather than skip counting Example inc 10s.

Repeat with 5 and 2

2, 5 and 10 times table charts available







Objective and strategy	Concrete	Pictorial	Abstract
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6  = 4  96 + 3 = 32	Continue to use bar modelling to aid solving division problems.	How many groups of 6 are in 24? $ 20 $ $ 20 \div 5 = ? $ $ 5 \times ? = 20 $
Division with arrays	Link division to multiplication by creating an array and thinking of the calculations that can be created.  Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences	Find the inverse of multiplication and division sentences by creating eight linking number sentences.  7 × 4 = 28 4 × 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 × 4 28 = 4 × 7 4 = 28 ÷ 7 7 = 28 ÷ 4

### 13 Division

Objective and strategy	Concrete	Pictorial	Abstract
Children use known and derived facts to divide a 2 digit number by a 1 digit number (eg I know 30 ÷ 3 = 10	10	Children can continue to use diagrams to help them divide numbers into equal parts.  12 ÷ 3 = 4	Children use short division to divide a 3 digit number by a single digit with no exchanging.  121 3 363
Children divide a 4 digit number by a 1 digit number using short division and whole numbers (no exchanging or remainders)			

### Y4 Division

Objective and strategy	Concrete	Pictorial	Abstract	
Divide numbers up to 4 digits by a one-digit number using the formal written method of short division	3 2  3 2  3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.	Begin with divisions that divide equally with no remainder.  2 1 8 3 4 8 7 2	<b>45</b>
	We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Children can also use bar models to represent division.  Encourage them to move towards counting in multiples to divide more efficiently.		Division

Objective and strategy	Concrete	Pictorial	Abstract
Division with remainders	Use known facts to teach division with remainders  14 ÷ 3 = 1 know 12 ÷ 3 = 4 and then 1 have a remainder of 2	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.	Complete the divisions using R to show a remainder. $28 \div 8 = 3 R 3$ $29 \div 8 = 3 \text{ REMAINDER 5}$ dividend divisor quotient remainde  Use bar models to show division $37 \div 3 = 3 R 7$ Move onto divisions with a remainder. $0 \cdot 6 \cdot 6 \cdot 3 \cdot 5 \cdot 5$

### Y5 Division

Objective and strategy	Concrete	Pictorial	ictorial Abst	
Divide numbers up to 4 digits by a two-digit number using the formal written method of short division, where the answer has up to 2dp.		Progress to expressing remainders as fractions/decimals  0 2 8 0 5 13 3 6 4 5 26 3 9 5 2 6 5 7 8		
Divide numbers up to 4 digits by a two-digit number using the formal written method of long division, where the answer has up to 2dp.		432 ÷ 15 becomes  2 8 r 12  1 5 4 3 2 3 0 0 1 3 2 1 2 0 1 2  Answer: 28 remainder 12	432 ÷ 15 becomes  2 8 1 5 4 3 2 3 0 0 15×20 1 3 2 1 2 0 15×8 1 2 15×8  Answer: $28\frac{4}{5}$	432 ÷ 15 becomes  2 8 · 8  1 5 4 3 2 · 0  3 0 \

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